## Trust in Waves

An introduction to packet radio with AX.25 and elliptic curve cryptography

https://brannon.online/wopr.pdf

## Data over Radio

Most people associate "radio" with voice transmissions, but from the beginning, radio has always been about transmitting arbitrary data from one place to another.

This started with with experiments in "wireless telegraphy" (which actually predate radio) that lead to standard protocols like Morse code. Today, we have a diverse range of digital radio networks like cellular CDMA + GSM, 802.11 WiFi, Bluetooth & BTLE, XigBee, GPS, Digital broadcast television, etc.

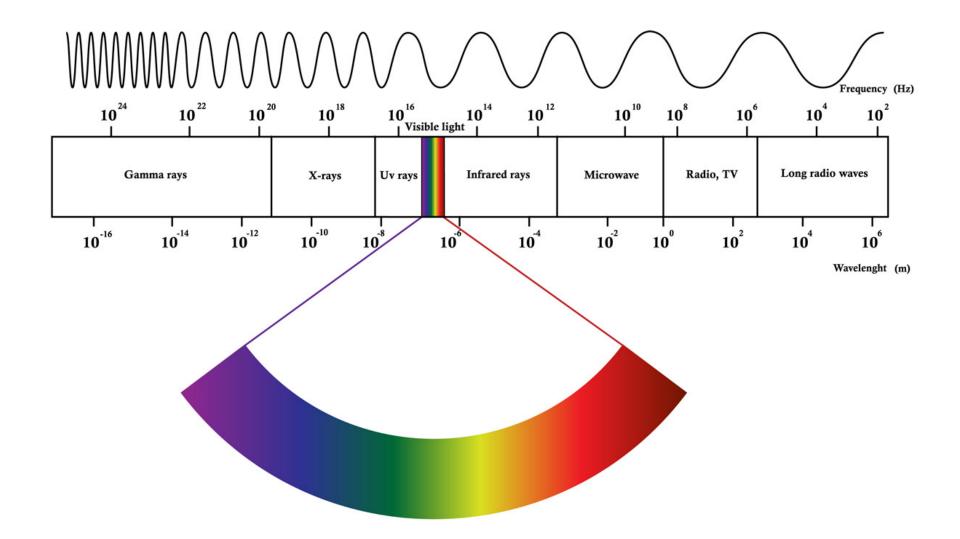
In this workshop, we'll be taking a look at some older, inefficient, and cheap means of transmitting data over radio by encoding it first as audio.

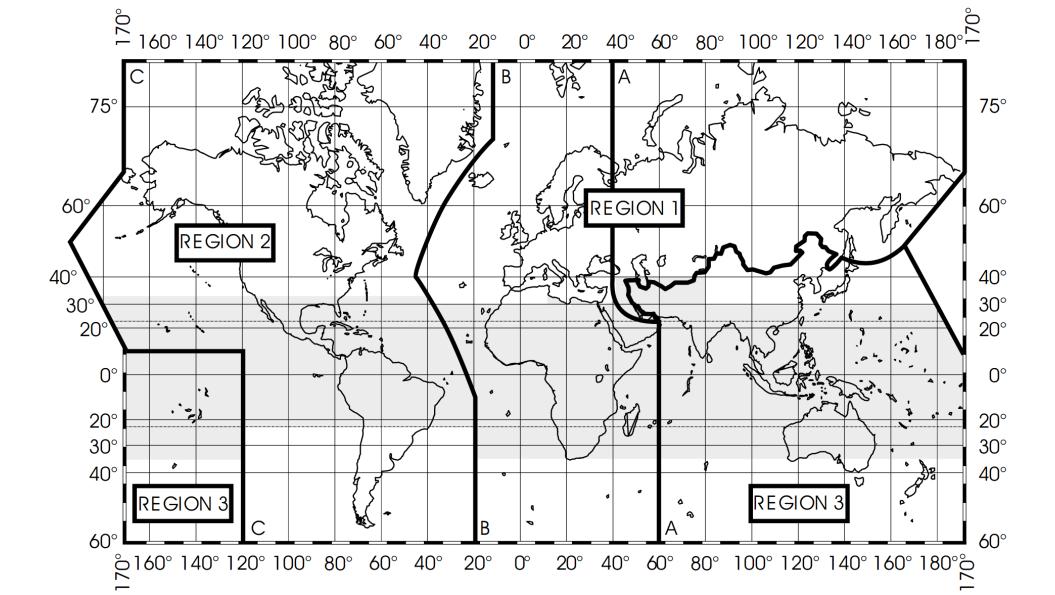
## Electromagnetic Spectrum

"Radio is the technology of using radio waves to carry information, such as sound and images, by systematically modulating properties of electromagnetic energy waves transmitted through space, such as their amplitude, frequency, phase, or pulse width."

- 469 editors on Wikipedia







### **UNITED**

### **STATES**

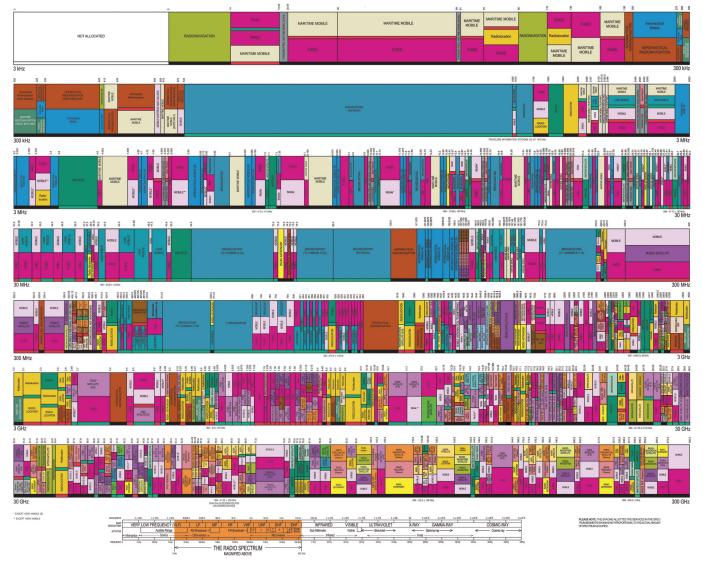
### **FREQUENCY**

### **ALLOCATIONS**

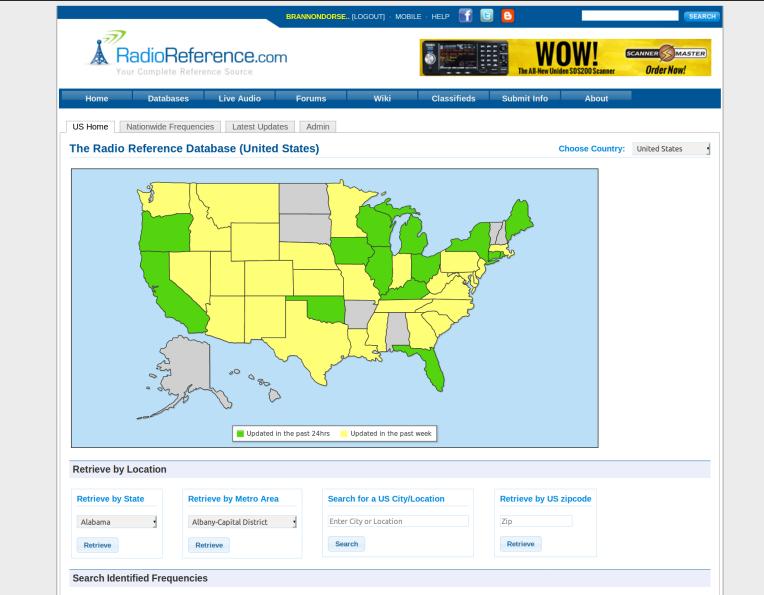
#### THE RADIO SPECTRUM



U.S. DEPARTMENT OF COMMERCE







### **Bally's/Wild Wild West**

560

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Α

**Bally Food** 

Bally H/K

Bally Trnsp?

Food

Transportation?

Housekeeping

|                | Syst    | em Nan       | ne: Bally                   | Bally's/Wild Wild West       |                   |                  |          |  |  |  |  |  |  |
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|                | Loca    | tion:        | Atlan                       | Atlantic City, NJ            |                   |                  |          |  |  |  |  |  |  |
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|                |         |              |                             |                              |                   |                  | '        |  |  |  |  |  |  |
|                |         |              |                             | Site                         | Name              | Freqs            |          |  |  |  |  |  |  |
| All Talkgroups |         |              | ns l                        | 001 (1)                      | Site-1            | 935.6375         | 936.     |  |  |  |  |  |  |
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|                | DEC     | HEX          | Mode                        | Alpha Tag                    | Descriptio        | n                |          |  |  |  |  |  |  |
|                | 112     | 007          | А                           | BallySlot112                 | Slots             |                  |          |  |  |  |  |  |  |
|                | 144     | 009          | Α                           | BallySlot144                 | Slots             |                  |          |  |  |  |  |  |  |
|                | 176     | 00b          | А                           | BallySlot176                 | allySlot176 Slots |                  |          |  |  |  |  |  |  |
| 240 00f A E    |         | Bally Eng    | Engineering                 |                              |                   |                  |          |  |  |  |  |  |  |
| 272 011 A E    |         | Bally Sec    | Security                    |                              |                   |                  |          |  |  |  |  |  |  |
| 336 015 A E    |         | BallySecSurv | Security - S                | Surveillance                 |                   |                  |          |  |  |  |  |  |  |
|                | 400     | 019          | А                           | BallySlotTch                 | Slot Technicians  |                  |          |  |  |  |  |  |  |
| 528 021 A      |         | BallyH/K Pub | Housekeeping - Public Areas |                              |                   |                  |          |  |  |  |  |  |  |
|                |         |              |                             |                              |                   |                  |          |  |  |  |  |  |  |

https://bit.ly/ballys-freqs

Business **Business** Business Security Security **Business Business** 

**Business** 

**Business** 

**Business** 

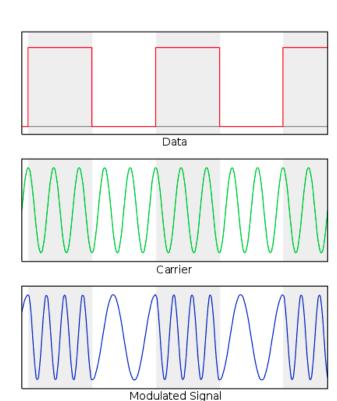
| All Tal | kgrou | ps 🕨 | 001 (1)      | Site-1      | 935.6375   | 936.725 | 937.1625c | 937.6625c | 937.700 | 939.225 | 939 | .2375    |
|---------|-------|------|--------------|-------------|------------|---------|-----------|-----------|---------|---------|-----|----------|
|         |       | '    |              |             |            |         |           |           |         |         |     |          |
| DEC     | HEX   | Mode | Alpha Tag    | Description | escription |         |           |           |         |         |     |          |
| 112     | 007   | Α    | BallySlot112 | Slots       |            |         |           |           |         |         |     | Business |

# Audio Frequency Modulation

Audio frequency-shift keying (AFSK) is a form of digital modulation that represents binary 1s and 0s by changes in the pitch of an audio tone.

AFSK defines the modulation technique but it doesn't define the transmission medium. Old telephone modems use the exact same type of AFSK that we'll use over radio waves.

Common bit rates for AFSK encoded data transmission of radio include 300, 600, and 1200 baud.



Encoding digital data as audio is slow, but it allows us to re-purpose existing systems that can receive or play audio, like cheap hand-held radios and walkie talkies.

All you need for a simple packet radio transmission setup is:

- 1. A cheap VHF/UHF radio like the Baofeng UV-5R.
- 2. A computer with audio input and output.
- 3. An audio cable that connects the computer's output to the radio's input and vice versa.



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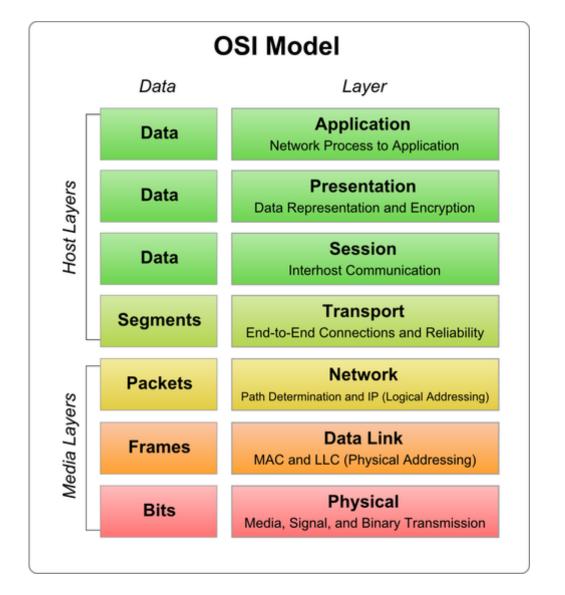
### Packet Radio

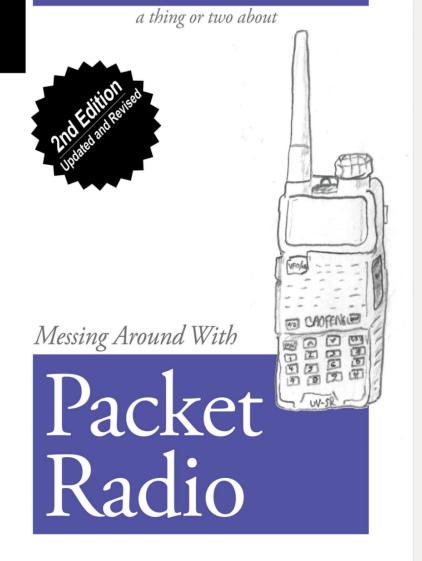
Packet Radio builds on top of digital modes like AFSK to group information into logical packets and frames, similar to TCP/IP. In fact, TCP/AX.25 is very common in the packet radio scene.

AX.25 (Amateur X.25) is the link-layer protocol of choice. It provides both *connected* and *connectionless* modes and uses amateur radio call signs as addresses.

| First Bit Sent |                               |        |        |          |         |          |  |  |  |  |
|----------------|-------------------------------|--------|--------|----------|---------|----------|--|--|--|--|
| Flag           | Address Control PID Info. FCS |        |        |          |         |          |  |  |  |  |
| 01111110       | 112/560 Bits                  | 8 Bits | 8 Bits | N*8 Bits | 16 Bits | 01111110 |  |  |  |  |

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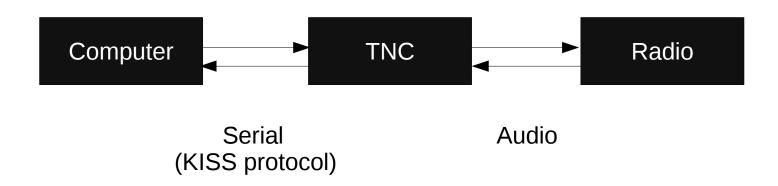
O'REALLY?

Dennis de Bel and Roel Roscam Abbing

# TNC

A *Terminal Node Controller* (TNC) is required for packet radio operation. The TNC is the modem that converts data to audio and vice versa.

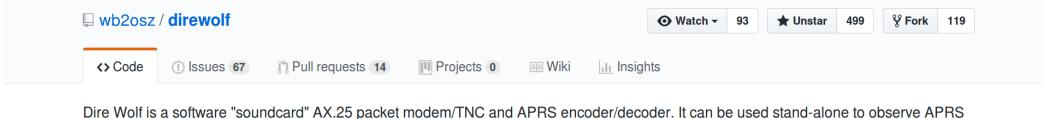
A TNC also acts like a network switch, assembling and dissasembling frames and packets. You send the TNC a packet of data, and it monitors the simplex channel and decides when to send that packet over radio.



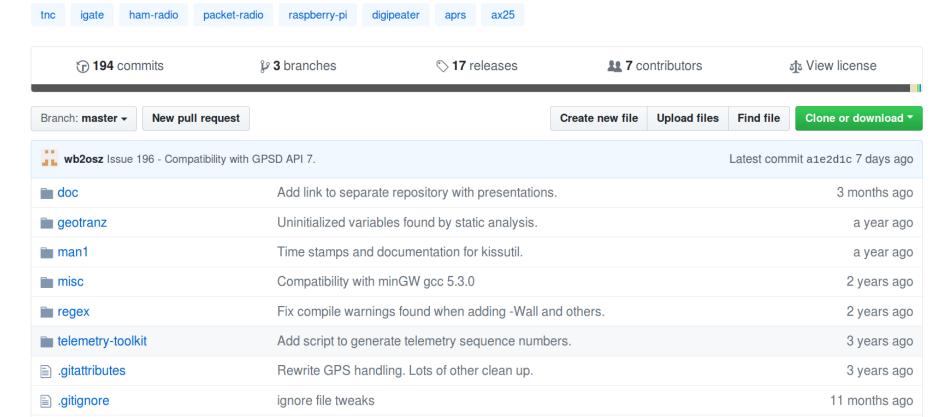
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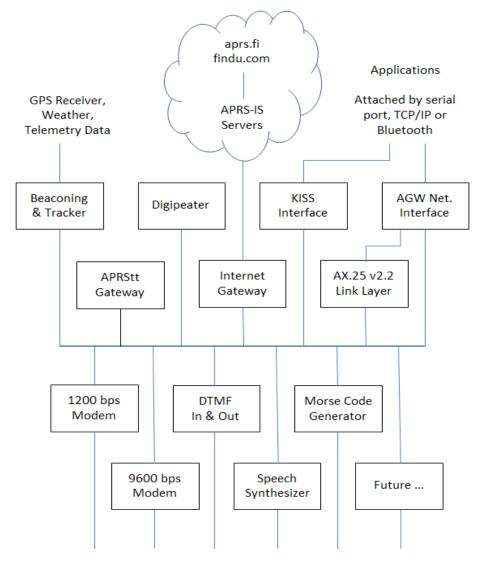






Dire Wolf is a software "soundcard" AX.25 packet modem/TNC and APRS encoder/decoder. It can be used stand-alone to observe APRS traffic, as a tracker, digipeater, APRStt gateway, or Internet Gateway (IGate). For more information, look at the bottom 1/4 of this page and in https://github.com/wb2osz/direwolf/blob/dev/doc/README.md



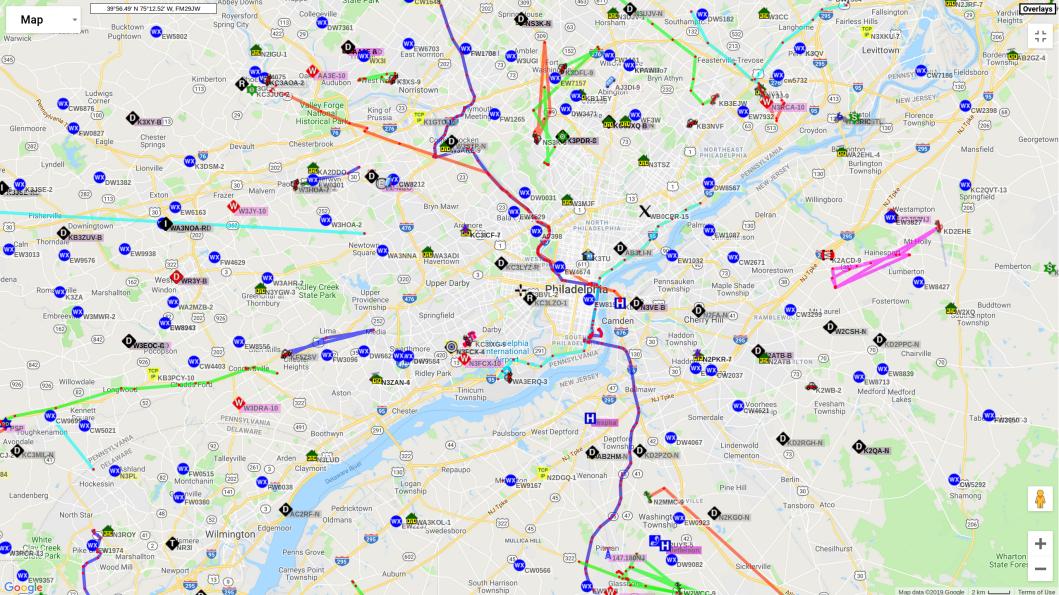


Traditional Radios & S.D.R.

## Automatic Packet Reporting System

A global packet radio network supporting GPS, weather station telemetry, text messages, announcements, bulletin boards and more. APRS data is often displayed on a map, showing stations, objects, tracks of moving objects, and direction finding data.

144.39 MHz 1200 baud

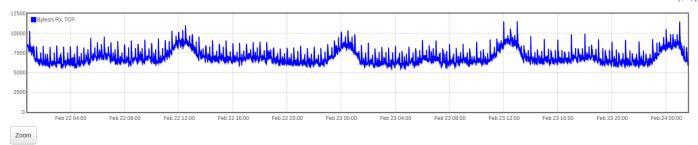


#### T2KA aprsc status 2019-02-24 01:38:38z



#### Server

| Server ID         | T2KA  |
|-------------------|---|
| Server admin      | Bernd Strehhuber, DM8BS                                 |
| Software          | aprsc 2.1.4-g408ed49                                    |
| Software features | epoll posix_cap clock_gettime gcc_atomics zlib ssl sctp |
| Uptime            | 83d5h   |
| Server started    | 2018-12-02 20:17:48z                                    |
| Operating system  | Linux i686  |



#### Totals

| Clients         | 249          | 0/s     |
|-----------------|--------------|---------|
| Connects        | 22151828     | 0.10/s  |
| Bytes Tx TCP    | 273056815218 | 32780/s |
| Bytes Rx TCP    | 49563191642  | 6307/s  |
| Packets Tx TCP  | 2677287144   | 329/s   |
| Packets Rx TCP  | 495021523    | 65/s    |
| Bytes Tx UDP    | 0            | 0/s     |
| Bytes Rx UDP    | 267196       | 0/s     |
| Packets Tx UDP  | 0            | 0/s     |
| Packets Rx UDP  | 2404         | 0/s     |
| Bytes Tx SCTP   | 0            | 0/s     |
| Bytes Rx SCTP   | 0            | 0/s     |
| Packets Tx SCTP | 0            | 0/s     |
| Packets Rx SCTP | 0            | 0/s     |

#### Duplicate filter +

| Duplicate packets dropped | 22374468  | 3.9/s |
|---------------------------|-----------|-------|
| Unique packets seen       | 470102173 | 62/s  |

#### Port listeners

| Address    | Name                                   | Clients  | Peak   | Max   | Connects   | Conn/s   | Packets Tx   | Packets Rx   | Bytes Tx   | Bytes Rx  | Tx/Rx bytes/s   |
|------------|--|--|--|---|--|--|--|--|--|---|---|
| [::]:14580 | Client-defined filter                  | 245  | 304  | 2000  | 22059429   | 0.10   | 693995054  | 31269576/20847300/2215811  | 77989898608  | 3832760900  | 8895 / 405  |
| [::]:14580 |  | 0  | 0  | 10  | 0  | 0  | 0  | 0/0/0  | 0  | 0   | 0/0   |
| [::]:10152 | Full feed                              | 4  | 12   | 100   | 92399  | 0  | 1974593910   | 664178/166601/3540   | 194214064186   | 147850054   | 23755 / 15  |
| [::]:10152 |  | 0  | 0  | 10  | 0  | 0  | 0  | 0/0/0  | 0  | 0   | 0/0   |
|            | [::]:14580<br>[::]:14580<br>[::]:10152 | [::]:14580 Client-defined filter<br>[::]:14580<br>[::]:10152 Full feed | [::]:14580 Client-defined filter 245 [::]:14580 0 [::]:10152 Full feed 4 | [::]:14580     Client-defined filter     245     304       [::]:14580     0     0       [::]:10152     Full feed     4     12 | [:]:14580         Client-defined filter         245         304         2000           [:]:14580         0         0         10           [:]:10152         Full feed         4         12         100 | [::]:14580         Client-defined filter         245         304         2000         22059429           [::]:14580         0         0         10         0           [::]:10152         Full feed         4         12         100         92399 | [:]:14580         Client-defined filter         245         304         2000         22059429         0.10           [:]:14580         0         0         10         0         0           [:]:10152         Full feed         4         12         100         92399         0 | [::]:14580         Client-defined filter         245         304         2000         22059429         0.10         693995054           [::]:14580         0         0         10         0         0         0           [::]:10152         Full feed         4         12         100         92399         0         1974593910 | [:]:14580         Client-defined filter         245         304         2000         22059429         0.10         693995054         31269576/20847300/2215811           [:]:14580         0         0         10         0         0         0         0/0/0           [:]:10152         Full feed         4         12         100         92399         0         1974593910         664178/166601/3540 | [:]:14580         Client-defined filter         245         304         2000         22059429         0.10         693995054         31269576/20847300/2215811         7798989608           [:]:14580         0         0         0         0         0/0/0         0/0/0         0           [:]:10152         Full feed         4         12         100         92399         0         1974593910         664178/166601/3540         194214064186 | [:]:14580 Client-defined filter 245 304 2000 22059429 0.10 693995054 31269576/20847300/2215811 77989898608 3832760900 [:]:14580 |

### Chattervox

Chattervox is a packet radio chat protocol with support for digital signatures and binary compression; think IRC over radio waves.

It's a new protocol with a reference implementation and command-line interface written in TypeScript.

In the United States, it's illegal to broadcast encrypted messages on amateur radio frequencies. Chattervox respects this law, while using elliptic curve cryptography and digital signatures to protect against message spoofing.

# FCC Title 47 Part §97.113 Prohibited transmissions

**Section (4)** Music using a phone emission except as specifically provided elsewhere in this section; communications intended to facilitate a criminal act; messages encoded for the purpose of obscuring their meaning, except as otherwise provided herein; obscene or indecent words or language; or false or deceptive messages, signals or identification.

# Packet Encapsulation

#### AX.25 Frame

Frame Type (UI)
Source Address
Destination Address
Payload:

### Chattervox Packet

Magic header
Packet Version
ECDSA Signature
Payload (compressed)

# Legal

It's illegal for non-licensed individuals to transmit on amateur frequency bands. There are, however, clauses that allow *unlicensed* individuals to speak, key/type, or otherwise transmit communication on behalf of a licensed amateur while they are under the direct supervision of a licensed control operator (*Third Party Traffic*). In other words, unlicensed peeps can group up with licensed folks.

Alternatively, the Multi-User Radio Service (MURS) provides 5 frequency channels in the VHF band that are *licensed by rule*, meaning anyone can use them without a license, provided they follow the rules.

#### Some MURS rules:

Blue Dot MURS-4, 154.570 MHz Green Dot MURS-5, 154.600 MHz 2 Watts Power Minimize interference Voice, data, image, telemetry allowed Repeaters & signal boosters NOT allowed

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# Frequencies

144.39 MHz **APRS**  145.0 MHz

Chattervox 1

145.5 MHz

**Chattervox 2** 

**APRS** 

CTVX 1

CTVX 2

MURS 4

MURS 5

154.57 MHz **Blue Dot MURS 4** 

154.6 MHz

**Green Dot MURS 5** 

# Baofeng UV-5R

The best (and only) RX/TX radio money can buy for \$25.

Made by a Chinese company that doesn't have to manufacture their radios to US standards.

Probably the most controversial radio on the scene. People either love them or hate them. In August 2018 the FCC issued citations to US distributors on Amazon.

Frequency Ranges:

136-174MHz (VHF) 400-520MHz (UHF)

Bandwidth:

25KHz on WIDE 12.5KHz on NARROW Power Output:

5W on HIGH 1W on LOW

Features:

VOX & PTT 128 Programmable Channels

Flashlight



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# VOX & Squelch

Simplex radios like the Baofeng can't receive and transmit at the same time. They can either be listening or speaking, but not both. All transmitters in a conversation share one frequency and they have to take turns speaking.

VOX stands for *Voice Operated Transmit*. It uses a radio's microphone to conditionally trigger transmit functionality as soon as input is detected. This is in contrast to *Push to Talk* (PTT) operation where the user (or computer) explicitly signals that they'd like to transmit.

Squelch sets a noise floor that must be broken by a transmitter in order for a receiving radio to sonify the signal.

If VOX is on squelch must be on as well.

# Installing Chattervox

Chattervox is currently fully supported on Linux, *kind of* supported on MacOS\*, and not yet supported on Windows. Chattervox requires a software or hardware TNC to operate.

\*https://bit.ly/chattervox-macos

```
# clone, build, and install the Direwolf TNC
git clone https://github.com/wb2osz/direwolf
cd direwolf
make
sudo make install
make install-conf
# install node via the node version manager
curl -o- https://raw.githubusercontent.com/creationix/nvm/v0.34.0/install.sh | bash
source ~/.bashrc # or ~/.bash profile
nvm install v8 # install node version 8
# install chattervox
npm install -g --cli chattervox
```

```
pi@cherry: ~
pi@cherry:~ $ chattervox send
Welcome! It looks like you are using chattervox for the first time.
We'll ask you some questions to create an initial settings configuration.
What is your call sign (default: NOCALL)? KC3LZO
What SSID would you like to associate with this station (press ENTER to skip)? 2
Do you have a dedicated hardware TNC that you would like to use instead of direwolf (default: no)? no
  "version": 3.
  "callsign": "KC3LZO",
  "ssid": 2,
  "keystoreFile": "/home/pi/.chattervox/keystore.json",
  "kissPort": "/tmp/kisstnc",
  "kissBaud": 9600,
  "feedbackDebounce": 20000
Is this correct [Y/n]? y
Generating ECDSA keypair...
Public Key: 04880e488c96d7fb55e7070dc46328fa206bbcacff9f5aa5dccdfd5a9aaf2591ba152ed751875cb593cec947866f4ad579
Settings saved to /home/pi/.chattervox/config.json
Error opening a serial connection to KISS TNC that should be at /tmp/kisstnc. Are you sure your TNC is running?
If you have direwolf installed you can start it in another window with "direwolf -p -q d -t 0"
pi@cherry:~ $
```

Includes optional support for: cm108-ptt Reading config file direwolf.conf Audio device for both receive and transmit: plughw:1,0 (channel 0) Channel 0: 1200 baud, AFSK 1200 & 2200 Hz, E+, 44100 sample rate / 3. Note: PTT not configured for channel 0. (Ignore this if using VOX.) Ready to accept AGW client application 0 on port 8000 ... Ready to accept KISS TCP client application 0 on port 8001 ... Virtual KISS TNC is available on /dev/pts/2 Created symlink /tmp/kisstnc -> /dev/pts/2 [OL] KC3LZO-2>CO:z9<0x01><0x02>705<0x02><0x18>=n}o<0x17><0x03>#.0x02><0x19><0x00>0x0 a>)6atla<0x0d><0x1c>.UThis is an example chattervox message! [0L] KC3LZO-2>C0:z9<0x01><0x02>705<0x02><0x19><0x00><0x07>z<0x14><0x17>zKx'<0x1c>5<0 x02><0x18>2b?Dm4<0x16>DGI0x1b>0x0c>WDYou'll see that longer messages usually get com pressed. [0L] KC3LZ0-2>C0:z9<0x01><0x03>604<0x02><0x18>s?<0x0e>0x0a>qrV<0x13>n dJ%7Rw<0x02><0x 18>P<0x12><0x08>HNh!i]<0x13>G<0x13><0x01><0x7f><0x0b>,0KUH/H\*-0(,<0x06><0x0b><0x16>< 0x14>%&\*a((\$<0x16><0x14><0x16><0x17>0x01><0x00> [0L] KC3LZO-2>CO:z9<0x01><0x03>604<0x02><0x18>F<0x1c>K<0x02><0x18><0x0d>lk~Mbv<0x18> OKs-(J-.JTN.-\*VS(U0x05>J%\*d<0x16>+<0x14><0x16><0x01>U%\*\$C0x01><0x00>

[OL] KC3LZO-2>CQ:z9<0x01><0x03>604<0x02><0x18><0x09>G1<0x13>.S?<0x1a>jpz<0x02><0x1e> <<u>0x02><0x18>{<0x00>ri</u><0x1d>!<0x17>gk<0x18><0x1d>0x1a>~x-\*NUH,)I-\* <0x00>2S<0x15>K<0x

pi@cherry:~ \$ direwolf -p -q d -t 0

14>J0x15><0x0b>R<0x15>j<0x15>rT=<0x00>

Dire Wolf version 1.5 (Feb 16 2019) Beta Test 4

KC3LZO-2: This is an example chattervox message!
KC3LZO-2: You'll see that longer messages usually get compressed.
KC3LZO-2: That one didn't, but this one probably will get compressed.
KC3LZO-2: Compression only occurs if the message is smaller once compressed.
KC3LZO-2: Otherwise chattervox chooses not to compress the message.
KC3LZO-2: ■

[0] 0:node\*

## Basic Usage

# open the chat room chattervox chat

# send a packet from the command-line chattervox send "this is a chattervox packet sent from the command-line."

# receive \*all\* packets and print them to stdout chattervox receive --allow-all

# generate a new public/private key pair, and use it as your default signing key chattervox genkey --make-signing

# add a friend's public key to your keyring, so that chattervox can verify their messages chattervox addkey KC3LZO \ 044da0d4c38bed6e5bc418231cb2dca4f690d858d36c38a032732553b76262a1adfccf588b6c1f9d7734b1bbce90914f82

# remove a friend's public key if it has become compromised chattervox removekey KC3LZO \ 0489a1d94d700d6e45508d12a4eb9be93386b5b30feb2b4aa07836398781e3d444e04b54a6e01cf752e54ef423770c00a6

# print all keys in your keyring chattervox showkey

# Chattervox Key Registry

Discussion on GitHub issues lead to the creation of a centralized "key server" where hams can register and share their keys via a secure channel.

For now, keys are added via pull requests to the chattervox-keys repository. (https://bit.ly/chattervox-keys)

The list of active and revoked keys is maintained and hosted on GitHub.

Future versions of Chattervox may allow you to automatically sync your local key store with these lists. Non-centralized key servers may also be explored in the future.

# Chattervox Examples

I've created a collection of example applications and use cases for the Chattervox protocol @ https://bit.ly/chattervox-examples

**Low-Fi Time Server**: Broadcast a time stamp beacon at regular intervals **A weather broadcast station**: Pulls local weather data from the Internet and broadcasts it via Chattervox

**Breaking news headlines**: Pulls breaking news headlines from the Internet and broadcasts them via Chattervox (technically illegal on Amateur Frequencies)

Remote shell: Use Chattervox to control a remote computer via Bash

**Zork**: Play the famous text adventure game over packet radio

| Byte Offset                    | # of<br>Bits | Name                      | Value  | Description  |  |  |  |
|--------------------------------|--------------|---------------------------|--------|--|--|--|--|
| 0x0000                         | 16           | Magic Header              | 0x7a39 | A constant two-byte value used to identify chattervox packets.   |  |  |  |
| 0x0002                         | 8            | Version Byte              | Number | A protocol version number between 1-255.   |  |  |  |
| 0x0003                         | 6            | Unused Flag<br>Bits       | Null   | Reserved for future use.   |  |  |  |
| 0x0003                         | 1            | Digital<br>Signature Flag | Bit    | A value of 1 indicates that the message contains a ECDSA digital signature.  |  |  |  |
| 0x0003                         | 1            | Compression<br>Flag       | Bit    | A value of 1 indicates that the message payload is compressed.   |  |  |  |
| [0x0004]                       | [8]          | [Signature<br>Length]     | Number | The length in bytes of the digital signature. This field is only included if the Digital Signature Flag is set.                                      |  |  |  |
| [0x0004 or<br>0x0005]          | [0-2048]     | [Digital<br>Signature]    | Bytes  | The ECDSA digital signature created using a SHA256 hash of the message contents and the sender's private key.  |  |  |  |
| 0x0004-0x104                   | 0-∞          | Message                   | Bytes  | The packet's UTF-8 message payload. If the Compression Flag is set the contents of this buffer is a raw DEFLATE buffer containing the UTF-8 message. |  |  |  |
| l indicates an optional field. |              |                           |        |  |  |  |  |

[] indicates an optional field.

# of

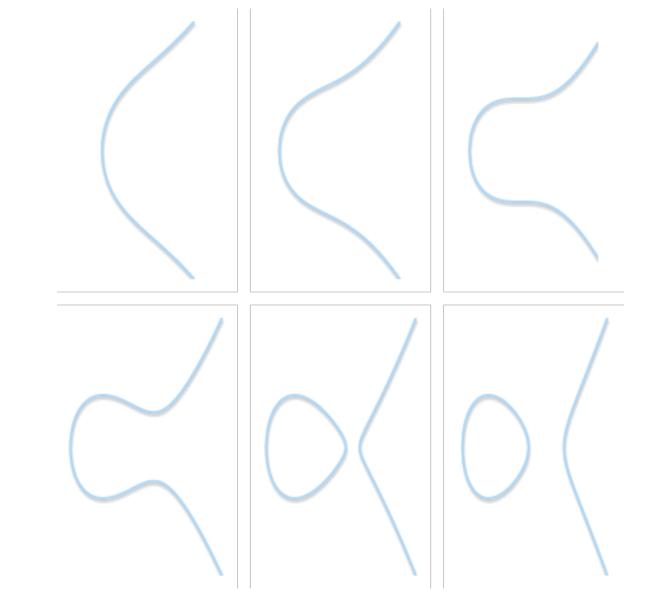
# What is an Elliptic Curve?

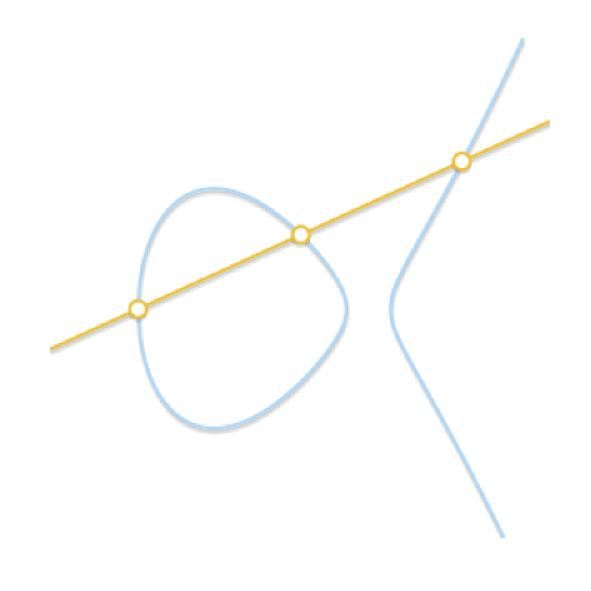
An elliptic curve is the set of points that are described by the equation...

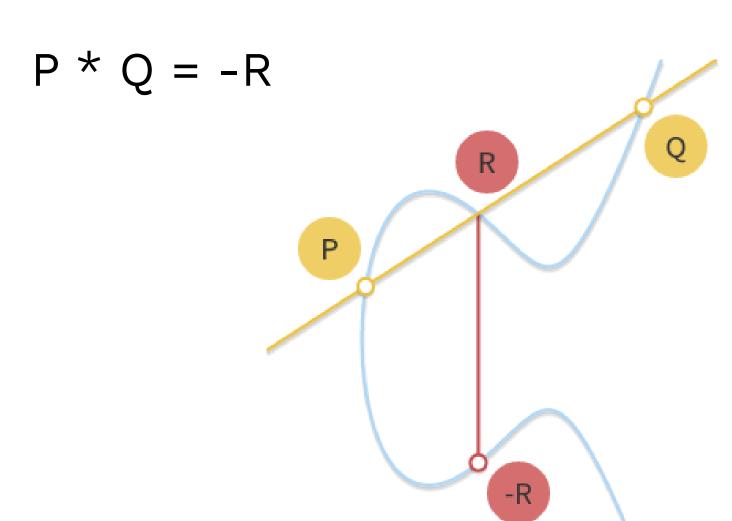
$$y^2 = x^3 + ax + b$$

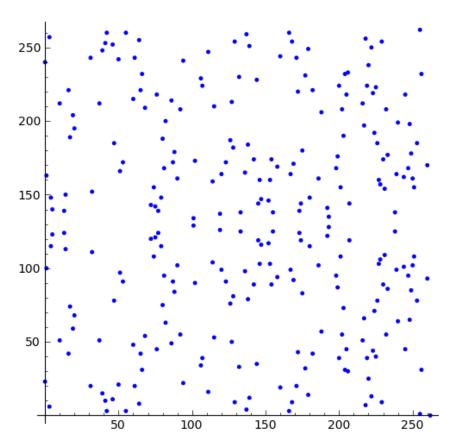
Elliptic curves are defined by two variables, **a** and **b**.

Curves are more general than functions and allow multiple y output values to exist for each x input value.









### STANDARDS FOR EFFICIENT CRYPTOGRAPHY

### SEC 2: Recommended Elliptic Curve Domain Parameters

Certicom Research

Contact: secg-talk@lists.certicom.com

September 20, 2000 Version 1.0

#### 2.5.2 Recommended Parameters secp192r1

The verifiably random elliptic curve domain parameters over  $\mathbb{F}_p$  secp192r1 are specified by the sextuple T = (p, a, b, G, n, h) where the finite field  $\mathbb{F}_p$  is defined by:

That's an "E" 😯

The curve  $E: y^2 = x^3 + ax + b$  over  $\mathbb{F}_p$  is defined by:

 $b \ = \ 64210519$  E59C80E7 OFA7E9AB 72243049 FEB8DEEC C146B9B1

E was chosen verifiably at random as specified in ANSI X9.62 [1] from the seed:

$$S = 3045 \text{AE}6F \text{ C8422F64 ED579528 D38120EA E12196D5}$$

The base point *G* in compressed form is:

$$G = 03 188DA80E B03090F6 7CBF20EB 43A18800 F4FF0AFD 82FF1012$$

and in uncompressed form is:

$$G=04\,188DA80E\,B03090F6\,7CBF20EB\,43A18800\,F4FF0AFD\,82FF1012$$
 07192B95 FFC8DA78 631011ED 6B24CDD5 73F977A1 1E794811

Finally the order *n* of *G* and the cofactor are:

$$h = 01$$

### **ECDSA**

The ECDSA Signature Algorithm creates a point (r, s) using a message, a private key  $(d_A)$ , and a **curve**.

$$e = SHA256(message)$$
 $z = left_bits(e, bit_length(n))$ 
 $k = random(1, n-1)$ 
 $(x, y) = k * G$ 
 $r = x \mod n$ 
 $s = k^{-1} * (z + r * d_A) \mod n$ 
 $signature = (r, s)$ 

The ECDSA Verification Algorithm verifies that **curve** point (r, s) was derived from a message, and the unknown private key  $(d_A)$  associated with a known public key  $(Q_A)$ .

```
e = SHA256(message)
z = left_bits(e, bit_length(n))
w = s^{-1} \mod n
u_1 = z * w \mod n
u_2 = r * w \mod n
(x, y) = u_1 * G + u_2 * Q_A

is_valid if r \equiv x \mod n
```

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Private Key (scalar) fccefae6a899e93b68c0ce7d6552449b6ef5b61ef0d26d78

Puplic Key (vector)

pubic\_key = curve\_base\_point \* private\_key

04427306E1725abb009991a132bc5a9346de5531915d8946d591fdfa8825bbc037ad8f818b043e6d9994ad58e64a405368

### Thanks

https://github.com/brannondorsey

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